

‘Epistemology of the line’ Reflections on the diagrammatical mind (2009)

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1. Thesis, Context and Assumptions

Diagrammatic inscriptions, among which we include graphic artefacts ranging from notations to diagrams to maps, are media that provide a point of linkage between thinking and intuiting, between the ‘noetic’ and the ‘aesthetic’. By means of this *interstitial graphic world*, the universal becomes intuitable to the senses and the conceptual becomes embodied: the difference between the perceptible and the intelligible is thus at the same time bridged – and constituted.¹ This is our thesis.

This thesis is embedded in a context which we shall briefly sketch here. Today, images are recognized as a legitimized object of research in epistemology and philosophy of science. They are considered not merely a means to illustrate and popularize knowledge but rather a *genuine* component of the discovery, analysis and justification of scientific knowledge.² However, the assumption that icons create evidence did not begin with our contemporary computer visualizations. Rather, the sciences have always been dependent on the ‘eye of the mind’, which gives the register of the senses access to the invisible, the abstract, the hypothetical and the imaginary.

The epistemic rehabilitation of iconicity takes place in the wake of talk of an ‘iconic turn’.³ This ‘iconic turn’ articulates a call to correct the claim to the absolutism of the linguistic made by the ‘linguistic turn’, as well as to bring to the fore the constitutive contributions of iconicity to our faculty of cognition and to the existence of epistemic objects. The upward revaluation of iconicity thus draws on a conceptual binary: the differentiation between language and picture.⁴ We are well acquainted with this differentiation, for which a wealth of related terms exists: discursive and iconic, telling and showing, representing and presenting, arbitrariness and resemblance, digital and analog, etc. Yet we are nonetheless confronted with a multiplicity of epistemically relevant phenomena that are not to be simply relegated to the side of language or the side of the image, which rather *conjoin qualities of both the linguistic and the iconistic*. This is the point of departure for the reflections to follow in this paper: there is a significant group of depictions that joins together traits of both language and imagery, albeit in varying proportions: Think of notation as language made visible; unutterable formal languages; music spatialized in the form of the score; diagrams that synthesize drawing with notation; or maps combining the digital

¹ Peirce explicitly assigned ‘diagrammatic reasoning’ this intermediary function between intuiting and thinking, as Stjernfelt (2000 and 2007) has shown. Other dimensions of this synthesis can, however, be detected in the work of other philosophers, as this essay and Krämer (2009) demonstrate in the cases of Plato, Kant, Peirce, and Wittgenstein. Stekeler-Weithofer (2008) has recently restored geometry as a diagram-based structural model to the centre of mathematics, thereby rehabilitating Kant’s synthetic a priori.

² As exemplified by Lynch (1998); Daston and Galison (2007); Bredekamp et al (2003); Bredekamp and Schneider (2006); Earnshaw and Wiseman (1992); Heintz and Huber (2001); Heßler (2006); Tufte (1997).

³ ‘Iconic turn’, see Boehm (1994, 13); ‘pictorial turn’, see Mitchell (1992).

⁴ Mitchell (1994, 5) pointed out, however, that there are no “‘purely’ visual or verbal arts” in the realm of the arts. See also Halawa (2008, 27ff.).

and the analog. The attribute shared by all these image-language hybrids is the *graphism of the line*. This graphism dwells on the far side of imagery but this side of language. The remarks to follow shall centre on a subsection of it that is here called ‘diagrammatics’. It is our goal to work out the *epistemological significance of diagrammatics*.

The knowledge-related functions of diagrams are not a new topic: publications shedding light on various facets of diagrams – primarily historically, but at times also systematically – have become abundant.⁵ To date, however, a *theory of diagrammatics* remains absent.⁶ It is precisely this missing theory to which we seek to contribute. Our ‘diagrammatic perspective’ is here connected with a historic-systematic re-framing that also concerns the extension of the concept ‘diagrammatics’. We presume that formalism, understood as an operative notation that uses the two-dimensionality of surfaces to depict non-visual matter by means of visual configurations and to operate syntactically with this matter, is also to be considered a modality of the diagrammatic.⁷ We thus include as diagrammatic not only diagrams in a narrow sense but *all* forms of intentionally created markings, notations, charts, schemes and maps. If this broadening of the concept ‘diagrammatics’ makes sense, the diagrammatic is a cultural technique⁸, which – in its historic implementations – forms an indispensable basis for virtually *all* activities of human cognizance. Two assumptions come into play here: (i) First, the anthropological supposition that the graphism of marks constitutes a defining feature of the human species. As a spatial-visual-tactile organizing form of thinking, arising from the coordination of eye, hand and mind, graphism is by no means secondary in significance to the cognitive role of language. (ii) Second, the supposition that not only sciences but even philosophy is twinned with diagrammatic structures. The diagrammatic features of the philosophical concept of reason remain largely neglected; reason, then, has yet to be reconstructed diagrammatologically. These assumptions form the backdrop before which we shall now investigate our thesis that diagrammatic artefacts act as a hinge between thinking and intuiting, while at the same time constituting the very differentiation between them.

We shall begin by elucidating two characteristic uses of diagrammatic operations in philosophy, Plato’s Simile of the Divided Line and Descartes’ coordinate geometry, as well as the figurative notation of Descartes’ *mathesis universalis*.

2. Plato’s Simile of the Divided Line

In *The Republic* (*Politeia*, 509 d – 511 e), Plato develops what is known as the ‘Simile of the Divided Line’, in which Socrates orders the ontological structure of the world according to an original-image relationship, and organizes this sub-categorization by degrees of knowability: one is to draw a line and divide it into two unequal subsections, such that the smaller subsection depicts the visible and the larger subsection the intelligible. These two subsections should then each be re-divided in

⁵ As exemplified by Anderson and Meyer (2001); Bogen (2005); Bogen and Thürlemann (2003); Châtelet (2000); Gehring and Keutner (1992); Gormans (2000); Jamnik (2001); Lüthy and Smets (2009); Schmidt-Burckhardt (2009); Siegel (2009); Stjernfelt (2007).

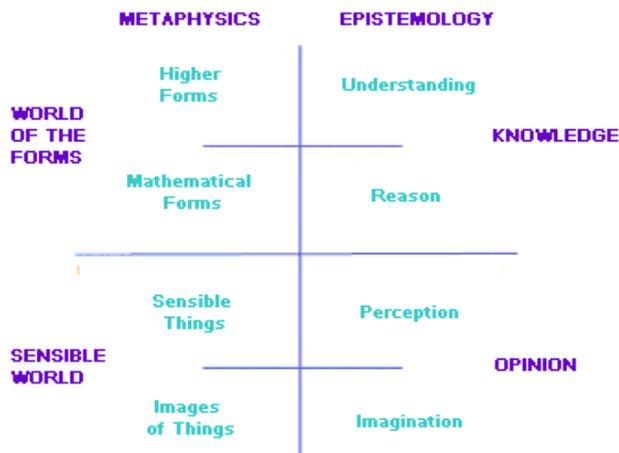
⁶ Lüthy and Smets (2009, 439); Mersch (2006, 103).

⁷ Cf. Krämer (2003); Krämer (2005)

⁸ On the concept of ‘cultural techniques’, see Krämer and Bredekamp (2003).

the same proportion as the original division. A four-part division of realms of being is thus created, which at the same time embodies a series of levels of cognitive knowledge – that is, the activities of the soul – with a progressively increasing degree of theoretical clarity.

Fig. 1 DEPICTION OF THE SIMILE OF THE DIVIDED LINE



http://www.molloy.edu/sophia/plato/divline_russo.gif

Within the region of the visible, the lowermost realm is comprised of mirror-images, shadows and reflections in water, which correspond to the epistemic state *eikasias*, that is, conjecture. The next section of the visible encompasses the originals of these copies, material objects such as inanimate objects, plants and animals. The cognitive activity corresponding to these is *pistis*, faith or belief. Together, these two levels form the domain of ‘doxa’, that is, opinion. In the third subsection, which opens up the realm of the intelligible and thus of the ‘episteme’, reside general concepts and mathematical objects. The form of knowledge here is *dianoia*, that is, thinking or understanding. The fourth section, in turn, is concerned with Forms as true being understood through *noesis*, the act of pure reason (that is, intellection), which for Plato constitutes the highest level of understanding.

We are unable here to undertake any analysis of the Simile of the Divided Line in terms of the multiplicity of its interpretations and unanswered questions.⁹ Rather, we wish to offer a summary of those aspects of the simile that make it a key moment for our understanding of diagrammatics. First, however, we must take a more precise look at the third level, where Plato locates mathematical objects, to him the protoform of scientific objects. As Plato characterizes this form of cognizance, mathematicians use visible objects as images representing invisible ideas: While mathematical speech and proofs refer to perceptible figures such as particular circles or numbers, they deal not with these concrete figures, but rather with the general concepts ‘circle’ or ‘number’, which are themselves not visible, but rather purely intelligible.¹⁰

⁹ Literature on the Simile of the Divided Line: Brentlinger (1963); Fogelin (1971); Nicholas (2007); Notopoulos (1936); Stocks (1911); Yang (2005).

¹⁰ Plato (1993, book vi, 510e): “They treat their models and diagrams as likenesses, when these things have likenesses themselves, in fact (that is, shadows and reflections in water); but they’re actually trying to see squares and so on in themselves, which only thought can see.”

The form of cognizance of *dianoia* treats the visible as the imaging for the senses of something that is purely intelligible. It is thus the distinctive feature of mathematical knowledge to depend indispensably on the sensory representation of its theoretical objects and at the same time always to remain conscious of the difference between the intuitive and the purely intelligible.

We want to stress four diagrammatologically instructive aspects.

(1) *Iconicity as ontological principle*: The capacity for making images – and with it visuality – forms the essence of Plato’s ontology, understood as a doctrine of what is real: even the highest level of being – the Forms – are introduced as originals, thus as templates for pictorial copies. Degrees of reality are held up against the measuring stick of the original-copy relationship. Correspondingly, Plato first introduced to philosophy the term *theoria*, which originally meant ‘Viewing of a festive performance’¹¹. In counterpoint to Plato’s perennially invoked hostility to images, to which philosophy’s hostility to images was able to casually attach itself, it must be noted that iconicity is the inner principle of organization of Platonic ontology and epistemology.

(2) *Differentiation between the visible and the intelligible; bridging of this differentiation*: Plato differentiates categorically between the perceptible and the intelligible, and thereby introduces a differentiation that was for 2000 years to form the lifeblood of philosophy. The Simile of the Divided Line inaugurates this differentiation. At the same time, however, it identifies an area of epistemic activity – *dianoia*, characteristic of mathematics and sciences – in which this difference is intentionally bridged in that sensory objects are recognized as images of the non-sensory. Aristotle himself designated these Platonistic objects, on which mathematical action focuses but does not linger, as ‘intermediary’ or ‘intermediate’¹². ‘Dianoia’, the activity of scientific reasoning, is – to express it in a modern way – to be understood as something conveyed by symbolic media. Even mathematicians need sensory representation in order to understand their noetic objects.¹³ Decisive here is the fact that the sensory objects deployed in thinking are neither a final product nor an end-stage, but rather are passed through en route to that which is not perceptible but only intelligible. The bridging function of the third level as an ‘interstitial world’ consists in its enabling of this movement of thought.

(3) *Spatiality of thinking*: Plato views cognizance as an activity characterized by an implicit spatiality. Thinking is directional and this direction can be characterized as an ascent, a rise through ascending levels. Thus is the Allegory of the Cave in the next book of the *Politeia* (*Republic*), a continuation of the Simile of the Divided Line by other means: It visualizes not abstract lines but rather the concrete situation of a cave, and knowledge is here imagined as an ascent out of the cave and into daylight.¹⁴ This inherent spatiality corresponds to the methodical matter that Plato, in the act of applying his own approach, deploys configurations of lines to visualize intellectual matters.¹⁵ For Plato himself, the diagrammatic scene becomes

¹¹ The original meaning of ‘theoros’ was the envoy of the polis sent to participate in festivals of the gods and oracles: see König (1998, 1128).

¹² Aristotle, *Metaphysics* I, 987b 14-17: “Further, besides sensible things and Forms there are the objects of mathematics, which occupy an intermediate position, differing from sensible things in being eternal and unchangeable, from Forms in that there are many alike, while the Form itself is in each case unique.”

¹³ On this interpretation, see Krämer (1991, 53ff.).

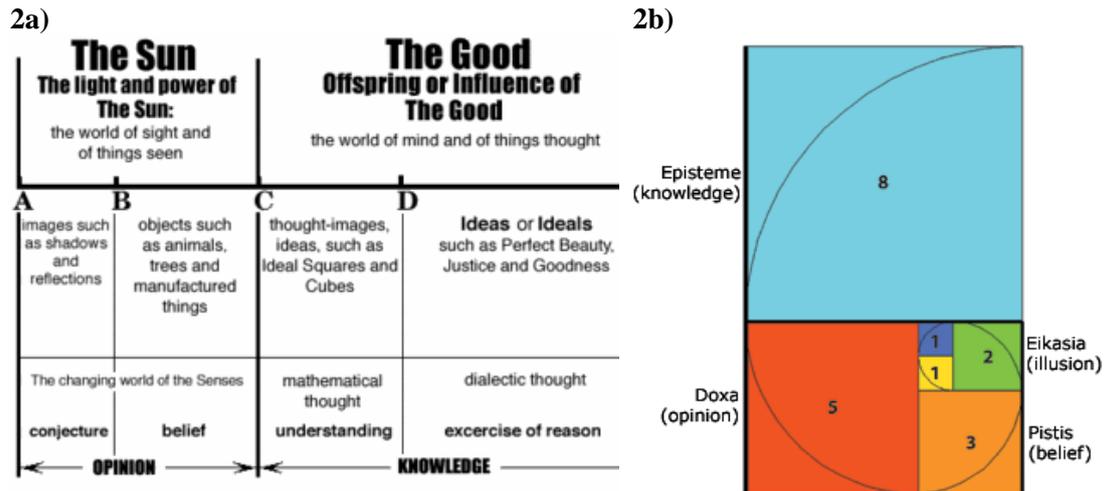
¹⁴ Plato, *Republic*, vol. vii, 514 -519.

¹⁵ Also instructive here is the use of geometric diagrams: Menon 82b-84c and 86e-87a, Theaitet 147c-148d, Politikos 266b.

the medium of insight. The diagram functions as an instrument of making evident the structure of ontology and epistemology.

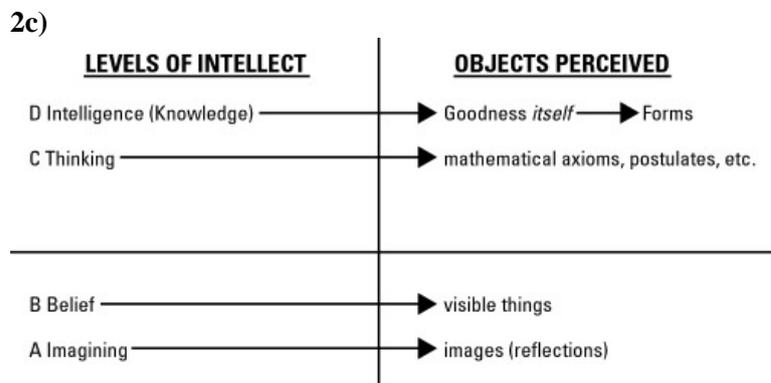
(4) *History of the reception of the Simile of the Divided Line*: While Plato's text has been handed down to us without any illustration, virtually all interpreters have supplemented the text with concrete diagrams. In scarcely any other case in all philosophy do diagrams appear as frequently in texts as in the secondary literature treating the Simile of the Divided Line.

Fig. 2 EXAMPLES OF DIAGRAMS OF THE SIMILE OF THE DIVIDED LINE



<http://media.photobucket.com/image/Platon%20analogy%20divided%20line/terrymockler/Plato-Divided-Line.gif>

http://www.inctr.org/publications/images/2004_v04_n04_s01b.gif



http://media.wiley.com/lit_note_images/154/1.jpg

Two features of these secondary visualizations are particularly striking: (i) The line that is to be subdivided is drawn in a variety of ways: horizontally or vertically, with the apportionment of the bottom subsections beginning on the left or the right. (ii) Equally multifarious and divergent are the interpretations of the Simile of the Divided Line.¹⁶ It would be interesting to examine the relationship between the variations in interpretations and the differences in corresponding visualizations of the Simile of the Divided Line. We cannot undertake such an investigation here. We would like, however, to venture a conjecture to which this history of reception leads us: a line is not simply a line. Diagrammatic depictions turn on the localization and direction of the line on the two-dimensional surface, as this sets the course for reading and

¹⁶ A compilation of various diagrams is found in Ueding (1992, 29ff.).

interpretation. At the same time, however, it is clear that diagrams do not interpret themselves.¹⁷ A diagram is always bound up with multiple interpretations. And inversely, the same thought can be visualized in a great variety of ways, even in the case of the simplest line diagram.

3. Descartes: Coordinate System, Analytical Geometry, and *mathesis universalis*

We shall now jump over to the philosophy and mathematics of the Early Modern Era. Here, Descartes (1596-1650) provides us with the second key moment for the diagrammatic. Since the discovery of incommensurability¹⁸ in ancient Greece, arithmetic and geometry, the countable and the measurable, had been treated as domains of mathematical objects to be vigilantly differentiated.¹⁹ Descartes now prompts a minor mathematical revolution in that by virtue of a two-dimensional coordinate system²⁰ he rejoins the geometric figure to the algebraic formula. It is not the use of coordinates as ‘reference lines’ per se that is so consequential but rather the use Descartes makes of the coordinates in that through these lines he makes geometry and algebra jointly depictable and mutually translatable. He thereby initiates ‘analytic geometry’.²¹ Descartes published ‘La Géométrie’²² as one of the three appendices²³ to his treatise on method, the ‘Discours de la méthode’²⁴, in order to use particular scientific disciplines as examples demonstrating the usefulness of his general method. There is thus a genuine link between Cartesian ‘coordinate geometry’ and Cartesian epistemology; our supposition is that the linear form of diagrammatic inscriptions plays a decisive role therein.

As was done with Plato, with Descartes we must limit ourselves to a roughly outlined reconstruction and *cannot* proceed historico-philosophically.

(1) *Coordinate geometry*. Aided by the introduction of a coordinate-like reference system – which, however, is in Descartes not yet right-angled – number pairs can be assigned to points and can with clarity be numerically localized in their position on a surface. Geometric construction and problem solving can thus be transcribed in algebraic form and carried out as arithmetic problems. Descartes moreover replaced the ancient existence criterion that a geometrical figure be constructible with a compass and ruler with its algebraic calculability: curves that can be algebraically transcribed are genuine parts of scientific geometry; the others remain outside the science of mathematics.

But the mathematical effects of analytic geometry are of far less interest to us here than its diagrammatic imprint. We start here from the depiction of the now-familiar right-angled Cartesian coordinate system, named after Descartes although he made use of neither right angles nor the negative abscissa (x-axis) and ordinate (y-axis) axes. Lines are shaped into a configuration in the form of intersecting axes. The axes are numbered in ascending order. Numbers generally have no place in space. The line segment, however, allows for their

¹⁷ Using a wealth of historical examples, Lüthy and Smets (2009) illustrate that diagrams are under-determined.

¹⁸ On this discovery, see Fritz (1965).

¹⁹ More precisely, see Krämer (1991, 32-43).

²⁰ The diagram of the coordinate system and its geographic and mathematical history reach back to antiquity. Rottmann (2008) has recently reconstructed the traces of this history. Nicole Oresme (ca. 1323-1382), Pierre Fermat (1601-1665) and later Isaac Newton (1643-1727) are likewise central to modern coordinate geometry.

²¹ Krämer (1989).

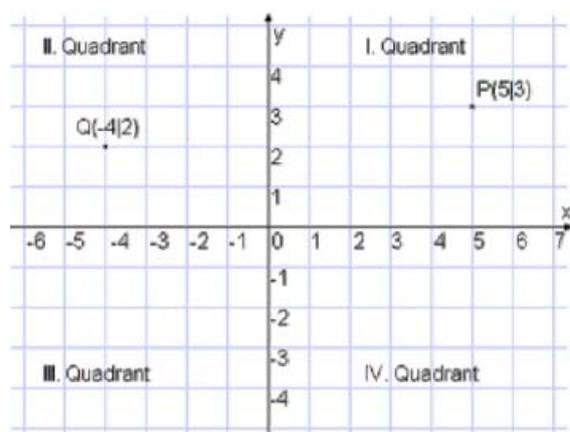
²² Descartes (1902).

²³ Descartes (1902): La Dioptrique, Les Météores, La Géométrie.

²⁴ Descartes (1902, 1-77).

simultaneous plotting and thus also placement. The axes have arrows, are thus directional, and subdivide the mathematical plane into four clearly defined and organized quadrants.

Fig. 3 THE RIGHT-ANGLED ‘CARTESIAN COORDINATE SYSTEM’, NAMED AFTER DESCARTES



http://de.wikipedia.org/wiki/Kartesisches_Koordinatensystem

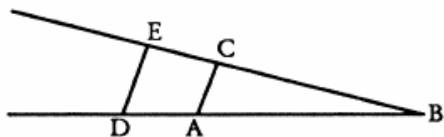
A fundamental diagrammatic insight becomes apparent here: the interaction of line and surface creates diagrammatic inscription's 'own space'. The arrangement of the axes enables a mathematical space to emerge from an empty surface. This space contains well-defined and delimited regions, the quadrants, in which the positions defined by the number pairs can be either occupied or unoccupied. Thus does a metamorphosis take place: through the inscription of coordinates, a sensorial apparent, real writing surface is transformed into a two-dimensional, mathematical, ideal plane. The points marked onto the writing surface have a perceptible extension, which is necessary to make possible the points' numerical identification and localization through coordinates. But on the ideal mathematical plane these points are without extension. Thus, we could say that the points realize a 'double existence' in coordinate space. They fulfil a double function as both a material trace of a marking (*punctum*, Lat. 'puncture') and an ideal construct of cognition. The coordinate system not only transforms an empirical surface into an abstract space but also creates a form of spatiality in which materiality and ideality, the intuitive and the intelligible, become intertwined. Hand, eye, and mind can work together on this diagrammatically produced surface and thereby first bring to life what we here shall call the 'eye of the mind'. In this sense, the coordinate system is a key to understanding diagrams in general.

(2) *The transformation of geometry into a language.* Coordinate geometry is a mechanism of translation: it makes two-dimensional geometric figures and linear algebraic equations mutually transferable. The term 'translation' has been chosen quite deliberately here: Descartes changes the very 'nature' of geometry in that he makes geometry 'compatible' with arithmetic. By virtue of the mediating position of the coordinate system, planarity turns into an identifying feature of analytic geometry, as it is based on the reduction of a three-dimensional geometry of solids to a two-dimensional plane geometry of line segments.²⁵ Descartes thereby turns geometry into a visual language, a transformation he is able to achieve by dispensing with the 'principle of homogeneity'.²⁶

²⁵ This is already proclaimed in the first sentence of *La Géométrie*: "Any problem in geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for its construction."

²⁶ More precisely: Krämer (1989).

Fig. 4 GEOMETRIC CONSTRUCTION OF MULTIPLICATION



“For example, let AB be taken as unity, and let it be required to multiply BD by BC. I have only to join the points A and C, and draw DE parallel to CA; then BE is the product of BD and BC.”

The Geometry of René Descartes, trans. from French and Latin by David E. Smith and Marcia L. Latham, New York: Dover Publication, 1954, 5.

The principle that two intersecting line segments determine a plane and two intersecting planes determine a solid had held since Greek antiquity. Descartes breaks from this principle: just as a number is created by the multiplication of numbers, the multiplication of line segments likewise results in a line. Descartes thus discards the ‘figurative significance’ of lines as structural elements of geometric figures and treats line segments as uninterpreted basic characters in calculations. Geometry becomes line calculation and Descartes’ ‘La Géométrie’ marks the birth of a *language-centred* understanding of the mathematical. Herein lies a key to the surge in quantification that occurred in the Early Modern Era: mathematics is no longer a realm of timeless, ideal objects, but rather advances to a type of universal and operative visual language, capable of expressing all that is quantifiable.

(3) *Mathesis universalis: The constituting of scientific objects through mathematical language.* It is in fact not until the posthumously published early form²⁷ of his treatise on method, the ‘Regulae ad directionem ingenii’²⁸, that Descartes formulates a novel science that he names *mathesis universalis*, for which he sees forerunners in both ancient problem-solving analysis and the new symbolic algebra developed by Viète.²⁹ The *mathesis universalis* is a science of all quantifiable objects. Its particularity is that it works with an artificial system of notation in the form of a two-dimensional graphism of extensional figures, a medium through which anything that is to be made an object of this science must be depictable. This figurative ‘language of the eye’ is operative: it not only represents its objects but also makes them manipulable and thus *generates* and *constitutes* them, as the unity of objects arises from the unity of the operation.³⁰ This is demonstrated by Descartes’ newly developed concept of a ‘universal quantity’, introduced as a reference object of his ideographic notation. As a result of the division between geometry and arithmetic, ‘quantity’ had always been subdivided into ‘magnitudo’, the geometric realm of the measurable, and ‘multitudo’, the realm of the arithmetically countable; there was no concept of a ‘universal quantity’. But Descartes created as the reference object of his *mathesis* precisely this concept of quantity in general, ‘magnitudo in genere’, which applies to everything extended, whether countable or measurable. Thus does the abstract concept of ‘universal quantity’ – seen with the ‘eye of the mind’ – receive a clear foundation in Cartesian thinking: on the one hand, in operations of the translatability of geometry and arithmetic, and on the other hand in the representability of quantifiable objects in

²⁷ On the publishing history see Descartes (1902, 351-396).

²⁸ Descartes (1902, 359-369).

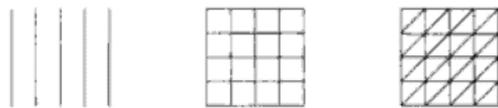
²⁹ Rule 4, Descartes (1904, 376).

³⁰ This, incidentally, becomes the core concept of the idea of method in Descartes’ epistemology.

the extensional figurative language of the *mathesis universalis*. The ‘*magnitudo in genere*’ is created in the act of being brought to intuition.³¹

(4) A last step remains to be taken, since it can be shown that the attributes of the material world, the ‘*res extensa*’ (corporeal substance), are originally derived from the model of figurative extension. In the *mathesis universalis* of the early Descartes, ‘*figura et extensio*’ remain attributes of the symbolic system of depicting the material world; but in the classic ‘*metaphysical Descartes*’, figure and extension are promoted to fundamental traits of material reality itself. In the ‘*Regulae*’, the early Descartes initially grounds the privileged significance of ‘*figura*’ and ‘*extensio*’ in an argument rooted in his theory of perception.³² ‘Figure’ is that which can be touched and seen, that which can be easily conveyed and that which is so general and simple that it “is found in every object of sensory perception”³³ (Rule 12). All qualitative differences in perception (odours, colours, sounds) imprint themselves upon our optical apparatus in the form of two-dimensional graphic configurations. There is a sort of drawing pencil in the apparatus of perception. Descartes illustrates this reduction of qualities of perception to two-dimensional graphic figures by using three differently drawn rectangles, whose figurative differences are analogous to the differences in colour between white, blue and red.

Fig. 5 THE FIGURES DRAWN IN RULE 12 TO DESIGNATE DIFFERENCES IN COLOUR



Descartes, *Regulae ad directionem ingenii-Rules for the direction of the natural intelligence*, a bilingual edition, ed. G. Heffernan, Amsterdam-Atlanta: Rodopi, 1998, 142.

Let us now summarize our diagrammatologic reconstruction: (i) The diagram of a coordinate system does not illustrate anything, but is rather a constructive tool of demonstrating and problem solving. It opens up a visual as well as tactile space to operate and manipulate abstract, non-spatial entities made visible. What matters here is the operative functioning of the graphism of coordinates. The meaning of a diagram is in its technical use. (ii) The interplay of line and surface constitutes a space with a twofold character: a space for ‘observing’ and handling visual objects and a space for thinking about abstract objects. By virtue of this twofold nature, non-visual objects and theoretical entities can be spatialized, made intuitable and manipulable. This intermediary space corresponds to Plato’s third subsection of the line. (iii) The coordinate system fulfils the function of mediating and translating between heterogeneous spheres in two senses: first, it mediates between figure and formula, geometry and arithmetic, and thus gives rise to the concept of universal quantity; second, it connects intuiting with thinking without relinquishing the differentiation between them.

4. The line as a field of research and discourse:

So we see that amidst philosophical reflections in antiquity and the Early Modern Era, we come upon a form of reasoning that makes use of the ‘cognitive power of the line’. Numerous scholars have already engaged with the culture-endowing role of the line.

³¹ More precisely, see Krämer (1991, 201ff.).

³² Rule 12, Descartes (1904, 412ff.)

³³ Ibid. 413.

In the field of *cultural history*, for Tim Ingold³⁴ the production and pursuit of lines is ubiquitous in human activities such as walking, weaving, storytelling, singing, drawing and writing. Ingold develops a comparative anthropology of the line, in which he tracks the transformations of the line in concurrence with traces, threads and surfaces. Threads out of which surfaces can be formed and traces that are imprinted on solid surfaces constitute the two basic modalities of his archaeology of the line. The interaction of thread and trace, culminating in notational systems, shows itself to be the cohesion of weaving and writing, of texture and text.

In the field of *palaeontology*, André Leroi-Gourhan³⁵ examines the function of graphism: the early linear forms of ornament, hunting symbol and record-keeping arise from a graphic aptitude which Leroi-Gourhan sets alongside speech as equally fundamental in its world-constituting function. Language and graphism are not only on an equal par in their cognitive potential. Rather, the drawing and reading of symbols, in contrast to acoustic signal formation, was not practised anywhere before the emergence of homo sapiens.³⁶ Leroi-Gourhan detaches the graphic from the artistic image and the pictures of the arts. In its evolutionary origin graphism is associated with abstraction, not with mimetic concretion.

In the field of *art history* Wolfgang Kemp reconstructs the history of the concept 'disegno' in the sixteenth century. 'Disegno' proves to be a double-sided concept: it means both the graphé as everyday "origin and outset of all human activity" and the non-everyday genius of a divine plan inherent in nature. 'Disegno', therefore, "mediates actively between nature and art"³⁷. The art historian Horst Bredekamp draws upon Galileo's drawings of the moon, Darwin's coral-like diagrams of evolution, Mach's eye diagram, and Crick's spirals to analyse the cognitive power of the line: "On the border between thinking and materializing, these drawings and diagrammatic lines show a suggestive force all their own, which no other form of expression possesses... as the first trace of the corporeal on paper they embody thinking in its highest possible immediacy."³⁸ Bredekamp finds the line so essential to modern art and science precisely because it achieves a balance between intuiting and thinking, and thus – as previously set out in the term 'disegno' – plays a role in the sphere of the sensory as well as in that of the intellectual.

In the *study of literature* Georg Witte³⁹ offers a phenomenology of linear form and graphism in the interplay between the fluid performance of the writing or drawing hand and the contrasting stable figuration of lines. Witte examines the line in its double function: it is a heteropoetic medium of depiction and representation as well as an autopoetic creative power.⁴⁰ Between the line as an evidence-creating form of knowledge and as an aesthetic absolute lies a broad area, which artistic avant-gardes explore in practice and sound out in theory.

In *cognitive semantics* George Lakoff⁴¹ and Mark Johnson⁴², among others, demonstrate the ways in which spatial schemata metaphorically transfer bodily movements into cognitive domains. Thus does an implicit 'cognitive topology'⁴³ emerge, which reveals a nucleus of spatial orientation precisely in non-spatial, abstract thinking and in interactions with quantities. Alongside the 'container', 'interior/interior', and 'peripheral/central', the line proves to be a

³⁴ Ingold (2007).

³⁵ Leroi-Gourhan (1980)

³⁶ Leroi-Gourhan (1989, 238).

³⁷ Kemp (1974, 227); in contemporary theory of art there have, however, been attempts at a 'non-fixing' concept of the line: Busch (2007); Derrida (1997); Elkins (1995); Rosand (2002).

³⁸ Bredekamp (2002, 24).

³⁹ Witte (2007); on the phenomenological approach to the line see also Lüdeking (2006).

⁴⁰ Witte (2007, 37).

⁴¹ Lakoff (1988 and 1990).

⁴² Johnson (1987).

⁴³ Ibid. 29.

central topological scheme of classification – in conjunction with the ‘path’ as the way between a point of departure and a destination. Can we then view the drawn diagram as the exteriorization of our mental inner topology, which in turn results from an archaic metaphorization of originally external corporeal activities in space?

These cursory notes should suffice for us to pose a question: is it possible to construct an ‘epistemology of the line’ in the sense of a rehabilitation and reconstruction of the *explicit* and *implicit* diagrammatic structures of rational thinking? This question outlines the research programme of ‘Diagrammatology’. In continuation and revision of the present debate, this research programme seeks to shift two emphases:

(i) Frederick Stjernfelt⁴⁴ rehabilitated diagrammatical reasoning in his course-setting monograph ‘DIAGRAMMATOLOGY’, tracing the field back to the thinking of Charles Sanders Peirce with further reference to Edmund Husserl. Though Peirce explicitly shaped the concept of ‘diagrammatical reasoning’, he is by no means its starting point: Sciences and philosophy have in fact been marked throughout history since their inception by diagrammatic dimensions, which remain to be reconstructed in the work of Leibniz, Lambert, Kant and, alongside Peirce, of Wittgenstein and Deleuze. In particular, Derrida’s contribution must be acknowledged if ‘Grammatology’ is to be broadened into ‘Diagrammatology’.

(ii) In ‘THE PHILOSOPHICAL STATUS OF DIAGRAMS’, Marcus Greaves⁴⁵ sees the diagrammatical as historically fighting a rearguard action; it is, so to say, the loser on the battlefield of mathematical and logical strategies of symbolization. This interpretation is bound to Greaves’ placement of figure and formula, and ultimately also image and language, in binary opposition to each other. But as soon as formalism is grasped in its notational iconicity and as a hybrid between language and image, it forms a complementary diagrammatic modality to the intuitive figure. The diagrammatic then undergoes modification; it is not, however, displaced.

The remaining space shall only allow us to point out a few cognitive-technical and epistemological aspects of ‘diagrammatics in the spirit of an epistemology of the line’.

5. ‘Epistemology of the line’: Facets of a theory of diagrammatics

(1) Simultaneity: We assume a fundamental exteriority of the human mind,⁴⁶ which holds a diagrammatic dimension.⁴⁷ This dimension includes the use of notations, schemes, charts, tables, diagrams and maps for both practical and theoretical purposes. Regardless of the variety among these forms of depiction, they all share an essential attribute: they employ two-dimensional spatial configurations as the matrix and medium to depict theoretical matters and ‘objects of knowledge’. In contrast to the sequentiality of auditory and tactile impressions, seeing is grounded in simultaneity.⁴⁸ When our eyes are presented with things that are next to each other, we gain an overview; we can compare different sorts of things to see similarities and differences, as well as recognize relationships, proportions and patterns within the vast diversity. Diagrams make the dissimilar comparable.⁴⁹ Let us consider the Simile of the Divided Line: it is a *single* line through which Plato visualizes diametrically opposed realms of being. And it is the trick of the Cartesian use of the coordinate system to make the measurable

⁴⁴ Stjernfelt (2007).

⁴⁵ Greaves (2002).

⁴⁶ Koch and Krämer (2009).

⁴⁷ Also arising here is a connection to the idea of the ‘extended mind’, Clark and Chalmers (1998), and of the ‘embodied and embedded mind’, Clark (2008).

⁴⁸ Jonas (1997, 248ff.).

⁴⁹ For Jonas (1997, 259), seeing is thus among the five senses the privileged one for objectivity.

and the countable mutually transferable. The homogenization of the heterogeneous— incidentally, a root of all concept formation – is a key to the potential of the diagrammatic ‘handwork of the mind.’

(2) Own spatiality: This homogenizing function, which mediates between divergent things,⁵⁰ is possible to the extent that diagrammatic inscriptions span their ‘own spatiality’. This space exhibits three essential traits: it is (i) a two-dimensional surface, for the most part conceived without simulation of depth (e.g. central perspective); (ii) it is formatted and scaled, and thus evinces a directedness and a system of measure; (iii) it is composed as an interplay of inscription and blank space. All of this arises from the interaction of point, line and plane, the simplest product of which the schema⁵¹ is: schemata here understood as the possibility of presenting the shapeless as a shape. Lines constitute the archetypal form of clear shape-forming: they delimit and they exclude. Every mark on a surface creates an asymmetry that becomes the source of potential distinction: the circular line separates points within and outside of the circle; a line segment separates what is left or right of it, above or below it. Unlike the line, the point as an elementary form of the graphic is little investigated.⁵² The question of whether and how points are inserted and interpreted separates eras of civilization: we need only consider the laying out of numbers with the aid of calculi in Pythagorean pebble (psephoi) arithmetic, or the zero as the originating point of the coordinate system, or the end point reached in an oscillating movement before the direction of motion is reversed, or the vanishing point in the construction of perspective, or the dot shape of musical notes, or the punctuation marks used in many notational systems.

(3) Hybridity: Diagrammatic inscriptions join together traits of the iconic and the linguistic. They are characterized by the fusion of showing and telling.⁵³ In contrast to artistic images and spoken language they take a middle position in-between the two. The presumption of a disjunctivity between ‘pure’ image and ‘pure’ language proves phenomenologically mistaken: ‘image’ and ‘language’, iconicity and discursivity, are to be differentiated only as *concepts*; they form the opposite ends of a conceptual scale, between which all real symbolic phenomena reside in their respective mixtures of the two. In relation to diagrammatics, ‘iconicity’ has a threefold meaning: two-dimensionality, visuality, and trans-naturalistic resemblance. Discursiveness emphasizes three further attributes: syntacticity, referentiality and propositionality. To put it bluntly: diagrams are images that make assertions, and thus can be right or wrong.

In modern theories of the image, it was standard practice⁵⁴ to emphatically reject resemblance as a definiens of iconicity. But to the extent that diagrammatic inscriptions show what they tell, the principle of structural resemblance is fundamental to them; ‘structural resemblance’ in the same sense in which the formula of a circle resembles its concrete figure or the number of contour lines on a topographical map corresponds to the measured height of a mountain.⁵⁵

⁵⁰ Cf. Krämer (2008) on the role of a ‘third party’ as an entity of transmission and mediation, residing between heterogeneous systems as a fundamental definition of ‘mediality’. There is thus a genuine connection between media theory and theory of the diagrammatic.

⁵¹ It is thus no accident that Immanuel Kant’s ‘transcendental schematism’ (B 176 - B187) is illustrated through the example of the drawing of a line, and that he accentuates the line’s double character as “on the one hand being intellectual, on the other hand sensory” (B 178).

⁵² Schäffner (2003) is an exception.

⁵³ In all languages, however, showing is fundamental, just as images, conversely, are imbedded in texts.

⁵⁴ e.g. Goodman (1997).

⁵⁵ On this concept of a ‘trans-natural illustration’, see Krämer (2008, 311).

(4) Referentiality: Unlike the artistic image, which in the first line refers to itself, diagrammatic artefacts require an external point of reference: *something* is being shown. Diagrammatic inscriptions are transcriptions⁵⁶ and in this sense manuals or machines of translation. Referentiality, however, must not be naively understood as reference to a matter that exists independent of inscription. There are two reasons for this: (i) Only through the inner logic of diagrammatic spatiality is it possible to depict anything in a standardizing way, and that which is depicted is in this respect constitutively shaped by the medium. (ii) The ‘objects’ of diagrammatic depiction are always relations and proportions, which are not ‘inherent’ but are created by intellectual practices in the interaction of eye, hand and mind. Even topographical maps do not simply depict a landscape, but rather a *knowledge* of a landscape, according to the *methods of projection* that necessarily distort the three-dimensional.

(5) Operativity: Notations, diagrams and maps do not merely depict something; rather, they are used every day and for the most part unspectacularly, as cultural techniques. This aspect of their use shows a cognitive and a communicative dimension: (i) In cognitive terms, diagrammatic depictions open up a two-dimensional space for handling, observing and exploding the depicted. Whether sailors use a map to chart their course or chemists discover gaps in the inscriptional framework of chemical elements,⁵⁷ whether a musical score enables pieces of music to be analysed or composed in new ways, whether we solve arithmetic equations with the aid of algebraic formulas⁵⁸ or create artistic and scholarly texts through the formulation, deletion and rewriting of sentences:⁵⁹ the inscribed surface always creates a space of operations and the diagrammatic always functions as a tool and instrument of orientation, analysis, revision and reflection. (ii) In communicative terms, diagrammatic artefacts are, in their generally manageable format, thoroughly mobile. Not only do they serve to make something perceptible; that which is depicted can also be effortlessly transmitted, transported, circulated, reproduced and combined. As Bruno Latour puts it: “Inscriptions are not interesting per se but only because they increase the mobility or the immutability of traces.”⁶⁰

(6) Dependence: No line interprets itself.⁶¹ It is precisely Plato’s Simile of the Divided Line and the history of its reception that shows us so starkly that the same matter can be visualized diagrammatically in a variety of ways and, conversely, that the same diagram can elicit thoroughly divergent interpretations. Moreover, the meaning of historically transmitted diagrammatic artefacts is scarcely to be understood apart from the socio-cultural milieu of their usage or without situating them in a ‘tacit knowledge’ of graphic conventions and their interpretation. The diagrammatic is situated: whether in the context of the text in which it appears or – here consider architectural drawings and city plans – within practices. It comes as no surprise that the diagrammatic is in most cases a part of a cascade of transmissions and transcriptions.⁶²

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⁵⁶ On the theory of transcriptivity as constituting the transcribed, see Jäger (2002).

⁵⁷ Klein (2003 and 2005).

⁵⁸ Krämer (1988).

⁵⁹ Raible (2004).

⁶⁰ Latour (1990, 31).

⁶¹ Lüthy and Smets (2009).

⁶² Latour (1990).

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